Study the effect of spatial and temporal laser pulse profile on heating a semi-infinite target in spherical coordinates

by

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Abstract:- The Laplace integral transform technique in spherical coordinates is used in solving the problem of heating a semi-infinite target induced by surface absorption of a spatial and temporal laser pulse profile. Mathematical expression for the temperature distribution in the target is obtained, considering the cooling and the temperature dependent absorption coefficient of the irradiated surface. As an illustrative example computations were carried out on a semi-infinite target of (*Al*) target.

Keywords: Heating, Laser material Interaction, Laplace transform , laser pulse profile, spherical coordinates

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1 Introduction

The advantage of the laser as a light source with a very high power density and extremely small focus areas have a considerable interest in material processing such as cutting, drilling, surface hardening in engineering, medicine, etc. [1-9]. Due to this, the theoretical study of the thermal effects induced by a laser in solid target is very interest. This is because the information of how to control the heating, melting or evaporation processes or how to avoid it can be obtained.

Several authors have considered different aspects of these problems [10-27].In the previous works [9,13,17-19], only the effect of the temporal laser pulse on a semi-infinite target in one direction was studied. Also it is considered that the temperature distribution on the surface is homogenous.

The analysis of such problems of heating targets by a laser pulse are need to further studies to obtain formula which describe the thermal behavior for laser material interactions.

This study aims to solving the problem of the heating a semi-infinite target irradiated with a spatial Gaussian and temporal Ready [3] laser pulse profile in a spherical coordinates, considering surface absorption and cooling effect. This is accomplished through the mathematical expression for the temperature distribution in the target using the two dimensional Laplace integral transform technique for solving the heat transfer equation.

2 Computations

Assuming a laser pulse of arbitrary temporal and spatial distribution *g(r,t)* to be incident perpendicular to a semi-infinite target. The laser pulse should be has an enough energy to heating the target. It is assumed that the surface of the target is subjected to stream of air to cooling it. Neglecting the temperature dependence of the material of the target on the material parameters except, the absorption coefficient, which is assumed to be linearly dependent on the surface temperature. The relations governing the temperature distribution in a semi-infinite target in spherical coordinate with angle independent are given by:

1) Heat transfer equation in spherical polar coordinates if the heat transfer into the radial direction (r) of a semi-infinite target

$$\frac{\partial T(r,t)}{\partial t} = \alpha\left[\frac{\partial^2 T(r,t)}{\partial r^2} + \frac{2}{r}\frac{\partial T(r,t)}{\partial r}\right] \quad (1)$$

2) The **i**ntegrated heat balance equation

$$\int_0^\infty q_0\left[A_0 + A_1 T(0,\ t)\right] g(r,\ t) d(\ \pi r^2\ ) =$$

$$\frac{\partial}{\partial t}\int_0^\infty \rho c_p T(r,\ t) 2\ \pi r^2\, dr + \int_0^\infty h T(r,\ t)\, 2\pi \mathrm{rdr} \qquad (2)$$

3)The initial condition $T(r,0)=0$ (3)

4) The vanishing temperature as r goes to infinity

$$\lim_{r \to \infty} T(r,\ t) = 0 \qquad (3\text{-a})$$

Where $g(r,t)=g(t)\ \delta(\pi r^2)$ and $\delta(\pi r^2)$ is the Dirac delta function.

Where:

T(r,t) is the temperature distribution above ambient in the radial direction of a semi-infinite target, *r* is the vector which represents the distance between the point at which the temperature is calculated and the point at which the laser beam is incident, *t* is the time at which the temp. is calculated after initiating the heating process, ρ, c_p, α are the density, specific heat and the thermal diffusivity of the target respectively, q_o is the maximum laser intensity, A_0 and A_1 are the surface temperature independent and dependent constants of the target respectively, *h* is the heat transfer coefficient, *g(r,t)* is the relative temporal and spatial profile of the laser pulse of maximum value one, *T(0,t)* is the surface temperature above ambient of the irradiated target, where

$$\vec{r} = \vec{r}' + \vec{z}$$

r' is the vector which represents the distance from the incident point of the laser to any point on the surface which goes to ∞, $\vec{z}$ is the perpendicular distance originating from the surface ($z=0$) to infinity and $|\vec{r}|^2 = z^2 + r'^2$.

The Laplace transform technique with respect to time is applied on the eqn.(1) gives:-

$$s\tilde{T}(r,s) - \tilde{T}(r,\mathbf{0}) = \alpha\left[\frac{\partial^{\mathbf{2}}\tilde{T}(r,s)}{\partial r^{\mathbf{2}}} + \frac{\mathbf{2}}{r}\frac{\partial\tilde{T}(r,s)}{\partial r}\right] \qquad (4)$$

By using the condition (3) in the above eqn and applying the Laplace transform technique with respect to *r* on the both sides of eqn. (4) one gets:

$$-s\frac{\partial\tilde{\tilde{T}}(p,s)}{\partial p}=\alpha\left[-p^2\frac{\partial\tilde{\tilde{T}}(p,s)}{\partial p}-\tilde{T}(0,s)\right]$$

$$\therefore -\frac{\partial\tilde{\tilde{T}}(p,s)}{\partial p}=\frac{\tilde{T}(0,s)}{p^2-s/\alpha} \qquad (5)$$

By using the inverse Laplace transform with respect to *r* to eqn. (5) one gets:

$$\tilde{T}(r,s)=\frac{\tilde{T}(0,s)}{r\sqrt{s/\alpha}}\sinh\left(\sqrt{\frac{s}{\alpha}}r\right) \qquad (6)$$

Equation (6) can be written as

$$\tilde{T}(r,s)=\frac{\tilde{T}(0,s)}{r\sqrt{s/\alpha}}\left[\frac{e^{\sqrt{\frac{s}{\alpha}}r}-e^{-\sqrt{\frac{s}{\alpha}}r}}{2}\right] \qquad (7)$$

Applying the condition in eqn.(3-a) on the above equation when $r\to\infty$ the term $\exp\left(\sqrt{s/\alpha}\ \mathrm{r}\right)$ will goes to zero, so the solution does not give the appropriate solution, so we considered the second

solution which can be written as:

$$\tilde{T}_2(r,s)=\tilde{C}(r,s)\tilde{T}_1(r,s) \qquad (8)$$

Where:

$\tilde{T}_1(r,s)\equiv\tilde{T}(r,s)$ from eqn.(7).

By differentiating eqn. (8) twice with respect to *r* and substituting into eqn. (4) one gets:

$$r\frac{\partial J}{\partial r}T_{\mathbf{1}}+\mathbf{2}rJ\frac{\partial T_{\mathbf{1}}}{\partial r}+\mathbf{2}JT_{\mathbf{1}}=\mathbf{0}$$

Where $J=\dfrac{\partial C}{\partial r}$

By solving the above equations and substitute by the *C* value into equation 8, we obtain the total solution as follows:

$$\tilde{T}_T(r,s)=-\frac{\tilde{T}(\mathbf{0},s)}{r\sqrt{s/\alpha}}e^{-\sqrt{\frac{s}{\alpha}}r} \qquad (9)$$

Applying the Laplace transform with respect to *t* on eqn. (2) one get:

$$q_o\tilde{G}(s)=\mathbf{2}\pi\rho c_p s\int_{\mathbf{0}}^{\infty}\tilde{T}(r,s)r^{\mathbf{2}}dr + \mathbf{2}\pi h\int_{\mathbf{0}}^{\infty}\tilde{T}(r,s)rdr \qquad (10)$$

By substituting from eqn. (9) into eqn. (10)

$$q_o\tilde{G}(s)=-\mathbf{2}\rho c_p\int_{\mathbf{0}}^{\infty}\frac{s\tilde{T}(\mathbf{0},s)}{r\sqrt{s/\alpha}}e^{-\sqrt{\frac{s}{\alpha}}r}\pi r^{\mathbf{2}}dr$$

$$-\mathbf{2}\pi h\int_{\mathbf{0}}^{\infty}\frac{\tilde{T}(\mathbf{0},s)}{r\sqrt{s/\alpha}}e^{-\sqrt{\frac{s}{\alpha}}r}rdr$$

Solving the above equation gives the value of $\tilde{T}(0,s)$ as follows:

$$\therefore \tilde{T}(0,s) = \frac{q_o \tilde{G}(s)\sqrt{s}}{-2\rho c_p \pi \alpha^{\frac{3}{2}} - \frac{2\pi h \alpha}{\sqrt{s}}} = \frac{q_o \tilde{G}(s) s}{\left(-2\pi h \alpha \quad -2\pi \rho c_p \alpha^{\frac{3}{2}}\sqrt{s}\right)} = -\frac{q_o \tilde{G}(s) s}{a + b\sqrt{s}} \qquad (11)$$

Where $a = 2\pi h \alpha$ and $b = 2\rho c_p \pi \alpha^{\frac{3}{2}}$

By substitution from eqn. (11) into (9) and applying the inverse Laplace transform with respect to s one gets:

$$T(r,t) = \frac{q_o}{2\lambda\pi}\int_0^t \frac{G(\tau)}{r}\left[\begin{array}{l} \frac{1}{\sqrt{\pi}}\left(\frac{r}{2\sqrt{\alpha}(t-\tau)^{\frac{3}{2}}} - \frac{C_1}{\sqrt{t-\tau}}\right) e^{-\frac{r^2}{4\alpha(t-\tau)}} \\ + C_1^2 \, exp\left(\frac{rC_1}{\sqrt{\alpha}} + C_1^2 (t-\tau)\right) * Erfc\left(\frac{r}{2\sqrt{\alpha(t-\tau)}} + C_1\sqrt{t-\tau}\right)\end{array}\right] d\tau \qquad (12)$$

where $C_1 = \frac{h\sqrt{\alpha}}{\lambda} = \frac{a}{b}$,

$\tilde{G}(s) =$ l$_t${ $g(t)\left[A_o + A_1 T(\mathbf{0}, t)\right]$} and

$$G(\tau) = g(\tau)\left[A_o + A_1 T(\mathbf{0}, \tau)\right] \qquad (13)$$

Eqn.(12) gives the temperature distribution in a spherical coordinates for angle independent, when the heat source is a Dirac-pulse. For any spatial distribution of the laser pulse *g(r')*, Equation (12) should be convoluted with this profile as follows:

$$T(r',z,t)=\frac{q_o}{\lambda}\int_{0}^{t}\int_{0}^{\infty}\frac{G(\tau)g(r'-\rho)\rho}{\sqrt{\rho^2+z^2}}\left[\begin{array}{l}\frac{1}{\sqrt{\pi}}\left(\frac{\sqrt{\rho^2+z^2}}{2\sqrt{\alpha}(t-\tau)^{3/2}}-\frac{C_1}{\sqrt{t-\tau}}\right)\\ e^{\frac{-(\rho^2+z^2)}{4\alpha(t-\tau)}}\\ +C_1^2\exp\left(\frac{C_1\sqrt{\rho^2+z^2}}{\sqrt{\alpha}}+C_1^2(t-\tau)\right).\\ Erfc\left(\frac{\sqrt{\rho^2+z^2}}{2\sqrt{\alpha(t-\tau)}}+C_1\sqrt{t-\tau}\right)\end{array}\right]d\rho d\tau \qquad (14)$$

To obtained the *T(0, r′ ,t)* firstly we put in eqn.(14) $z=0$, $t\rightarrow\tau$ and $\tau\rightarrow u$, so one gets the Voltera integral equation. By using Neumann iteration method on the obtained Voltera integral equation, one gets the final form of *T(0, r′ ,t)* [21].

3 Results and discussions

Eqn. (14) has been calculated for a semi-infinite aluminum target subjected to both , a relative distributions of the Ready laser pulse profile [5] fitted by the following relation [3],

$$g(t)=\frac{(n+1)^{n+1}}{n^n}\frac{t}{\delta t}\left(1-\frac{t}{\delta t}\right)^n$$

for $0\leq t\leq\delta t$ *g(t)=0* *for* $t\geq\delta t$ and the spatial distribution of the Gaussian laser pulse profile $g(r')=\exp\left(\frac{(-4\ln 2)\,r'^2}{\Delta r'^2}\right)$ with maximum value one.

Where *g(t)* is temporal laser pulse profile at a time t with maximum value (1) and *n* is equal to 3, δt is laser pulse duration time and $\Delta r'$ is the spectral half width of the Gaussian laser pulse profile. Table 1 gives the thermal and optical parameters for aluminum target as an example.

Table 1. Gives the thermal and optical parameters for aluminum [28].

Element	λ $(W\,m^{-1}K^{-1})$	ρ (kg/m^3)	Cp $(j/kg.K)$	A_o	A_1 (K^{-1})	T_m (K)
Al-Slab	*238*	*2707*	*896*	*0.056*	$3.05x10^{-5}$	*633*

Table 2. Show the three variations of the surface absorption coefficient A_1 and the cooling coefficient h.

Case	A_1 (k^{-1})	h $W\,m^{-2}\,k^{-1}$
1	$3.05\ 10^{-5}$	0
2	0	0
3	0	10^5

Figures. 1,2 and 3 represent the surface temperature distribution as a function of time t calculated for ($r'=3.3\times10^{-4}$ m, $q_o=2\times10^{11}$ W/m^2). Considering temperature dependent absorption coefficient and cooling. These calculations are carried out for different pulse durations δt (1.5, 2, 2.5×10^{-5} sec.) respectively. From the Figures we show that, the temperature increases with increasing t up to maximum value and then decreases with the further increase of the time. This is because the absorbed laser power in the starting time is greater than the losses and the conducting heat power inside the target. As this phenomena is inverted at the end of the laser pulse, the surface temperature begins to decreases. The Figures show also that the effect of the cooling is very small this is because of the pulse duration times which are very small and leads to a small effect of the cooling process. Also we show that, the surface temperature is greater in case of $A_1 \neq 0$ than the other two cases. This behavior is due to the increased absorbed power by approximately 20 % than that in the case of $A_1=0$.

The Figures also show that, the greater the pulse duration time, the greater is the surface temperature distribution and vice versa. This is because the absorbed energy under the incident pulse profile increases with increasing the pulse duration time.

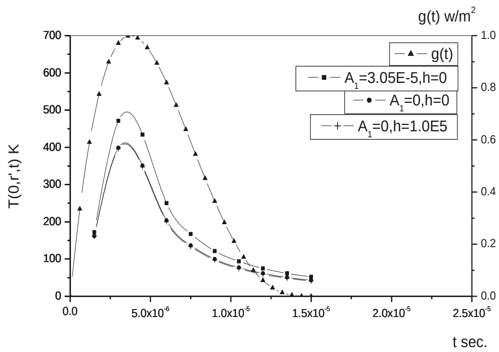

Fig.1. The time dependence of the surface temperature calculated during the irradiation of a semi-infinite *Al*-target with a spatial Gaussian distribution and with Ready laser pulse profiles g(t) with maximum value one , pulse duration $\delta t=1.5\times 10^{-5}$sec, intensity q_0=2.0E11 W /m^2, z=0 m and r' =3.3E-4 m for the cases listed in Table 2.

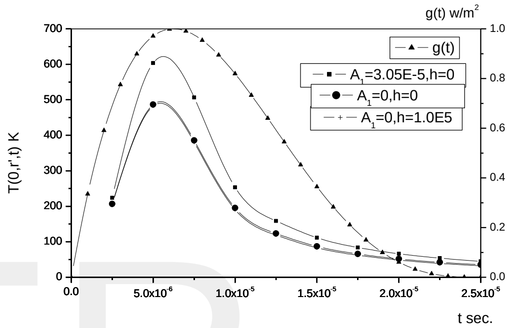

Fig.2. The time dependence of the surface temperature calculated during the irradiation of a semi-infinite *Al*-target with a spatial Gaussian distribution and with Ready laser pulse profiles g(t) with maximum value one , pulse duration $\delta t=2\times 10^{-5}$sec, intensity q_0=2.0E11 W /m^2, z=0 m and r'=3.3E-4 m for the cases listed in Table 2.

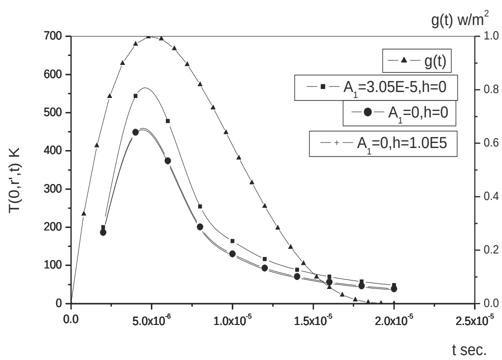

Fig.3. The time dependence of the surface temperature calculated during the irradiation of a semi-infinite *Al*-target with a spatial Gaussian distribution and with Ready laser pulse profiles

g(t) with maximum value one , pulse duration $\delta t=2.5\times 10^{-5}$ sec., intensity q_0=2.0E11 W /m^2, z=0 m and r' =3.3E-4 m for the cases listed in Table 2.

Figures. 4 and 5 show the effect of the incident power density *q(t)* on the surface temperature distribution calculated for q_o=(1.4, 1.6×10^{11} W/m^2) respectively. The calculations carried out at r'=3.3×10^{-4}m, δt =2.5×10^{-5} sec for all A_o, A_1, h values considered in Figs. (1,2,3). The Figures show the same behavior as shown in the Figs. (1,2,3). The Figures show as expected, an increase of the surface temperature with the increasing the power. Also in this case the cooling is found to have negligible effect on the surface temperature.

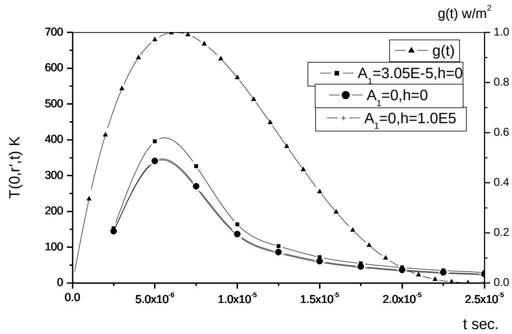

Fig.4. The time dependence of the surface temperature distribution calculated during the irradiation of a semi-infinite *Al*-target with Ready laser pulse profiles g(t) with maximum value one and pulse duration $\delta t=2.5\times10^{-5}$ sec., intensity q_0=1.4E11 W /m^2 ,z=0 m and r'=3.3E-4 m for the cases listed in Table 2.

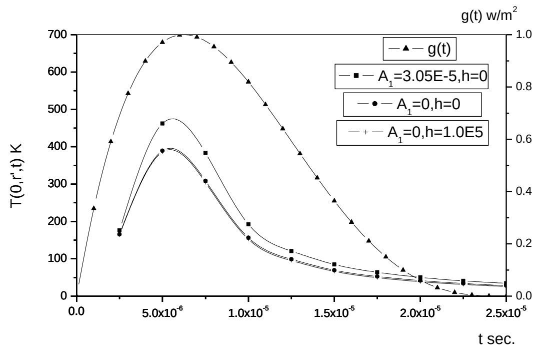

Fig.5. The time dependence of the surface temperature distribution calculated during the irradiation of a semi-infinite *Al*-target with Ready laser pulse profiles g(t) with maximum value one and pulse duration $\delta t=2.5\times10^{-5}$sec.,intensity q_0=1.6E11 W /m^2, z=0 m and r' = 3.3E-4 m. for the cases listed in Table 2.

Figures. 6,7 and 8 show the spatial temperature distribution calculated for different z-values at a distance $r'=3.3\times10^{-4}$ m from the maximum of the spatial Gaussian distribution of the laser radiation. The calculations are carried out for $t=2\delta t/10$ sec, $q_o=2\times10^{11}$ W/m^2 and pulse durations (1.5×10^{-5}, 2×10^{-5}, 2.5×10^{-5} sec.) and all considered cases of Figs. (1,2,3) respectively.

The figures show that, the greater the pulse duration, the greater is the thermal penetration depth. This is because by increasing the pulse duration the absorbed energy increases and so the surface temperature which allows enough energy to be diffused in the target. The figures show that while the existence of A_1 play a role in the absolute value of the temperature but not in the penetration depth, the cooling has practically no effect. The effect of A_1 can be attributed to the greater slope of the temperature which allows more heat to penetration in smaller region and so the slope reduces over small distances.

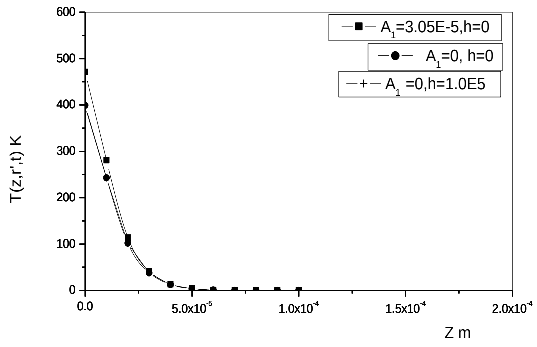

Fig.6. The temperature distribution in the z direction of a semi-infinite *Al*-target irradiated with a spatial Gaussian distribution calculated at $t = 3\times10^{-6}$ sec with Ready laser pulse profile g(t) considering $\delta t=1.5 \times10^{-5}$ sec. , intensity q_0=2.0E11 W /m^2 and r' =3.3E-4m, for the cases listed in Table 2.

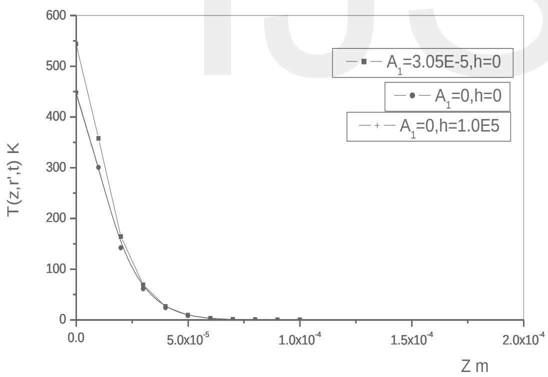

Fig.7. The temperature distribution in the z direction of a semi-infinite *Al*-target irradiated with a spatial Gaussian distribution calculated at $t = 3\times10^{-6}$ sec with Ready laser pulse profile g(t) considering $\delta t=2 \times10^{-5}$ sec. , intensity q_0=2.0E11 W /m^2 and r' =3.3E-4m. for the cases listed in Table 2.

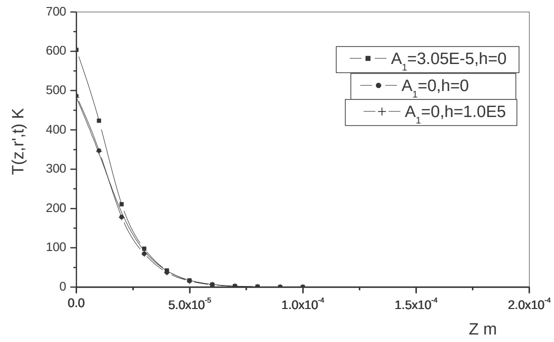

Fig.8. The temperature distribution in the z direction of a semi-infinite *Al*-target irradiated with a spatial Gaussian distribution calculated at $t = 3\times10^{-6}$ sec with Ready laser pulse profile g(t) considering $\delta t=2.5 \times10^{-5}$ sec. , intensity q_0=2.0E11 W /m^2 and r' =3.3E-4 m, for the cases listed in Table 2.

Figures 9 and 10 show the temperature distribution in the target as a function of z. the calculations are carried out at $t=2\delta t$ /10 , $r'=3.3\times10^{-4}$ m, δt =2.5×10^{-5} sec and different values of q_o (1.4×10^{11}, 1.6×10^{11} W/m^2). The Figures show the same behavior as in the previous cases of Figs.

6,7. The Figures show an increase of the temperature with increasing the power.

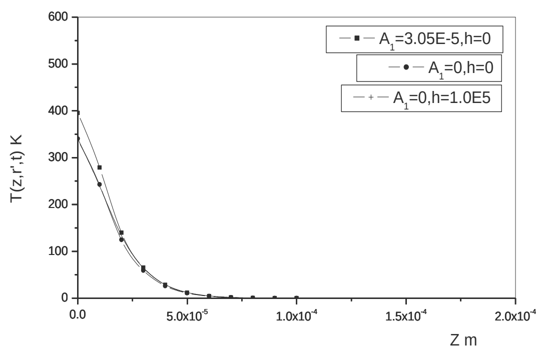

Fig.9. The temperature distribution in the irradiated semi-infinite *Al*-target calculated at $t = 3\times10^{-6}$ sec for the laser pulse profiles given in figure (1) considering $\delta t = 2.5 \times 10^{-5}$sec.,intensity q_0=1.4E11 W/m^2 and r' =3.3E-4 m for the cases listed in Table 2.

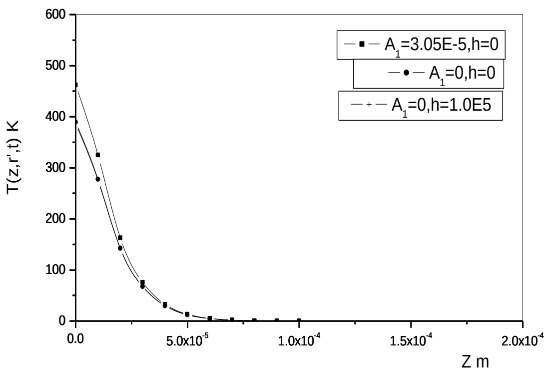

Fig.10. The temperature distribution in the irradiated semi-infinite *Al*-target calculated at $t = 3\times10^{-6}$ sec for the laser pulse profiles given in figure (1) considering $\delta t = 2.5 \times 10^{-5}$ sec., intensity q_0=1.6E11 W/m^2 and r' =3.3E-4 m for the cases listed in Table 2.

Figures 11 and 12 show the spatial dependence of the surface temperature distribution on the distance from the maximum of the Gaussian distribution of the laser radiation r'. The calculations are carried out for the considered *Ao*, A_1, and h values of Figs. 1,2,3, t=2 δt /10 sec and q_o equal to (1.4, 1.6×10^{11} W/m^2) respectively. From the Figures it is evident that the temperature tends to zero at practically the same distance but the surface temperature increases with increasing the laser power.

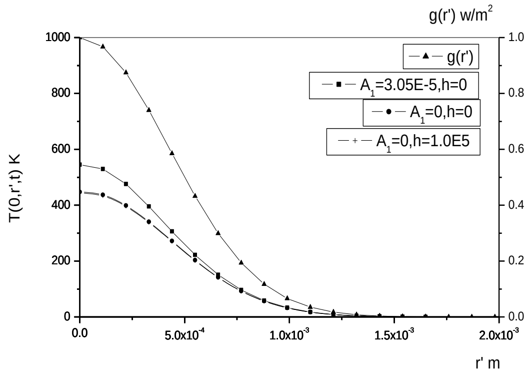

Fig.11. The spatial surface temperature distribution calculated during the irradiation of a semi-infinite *Al*-target with a spatial Gaussian distribution g(r') and with a Ready laser pulse profile g(t) with maximum value one , pulse duration $\delta t = 2.5 \times 10^{-5}$sec.,intensity q_0=1.4E11 W /m^2, and z=0 m for the cases listed in Table 2.

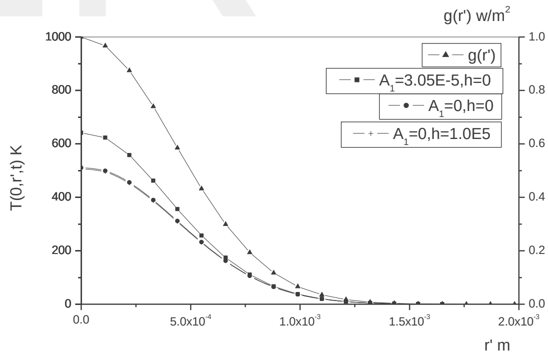

Fig.12. The spatial surface temperature distribution calculated during the irradiation of a semi-infinite Al-target with a spatial Gaussian distribution g(r') and with a Ready laser pulse profile g(t) with maximum value one , pulse duration $\delta t = 2.5 \times 10^{-5}$sec. ,intensity q_0=1.6E11

W /m^2, and z=0 m for the cases listed in Table 2.

4 Conclusions

The temperature dependence on the absorption coefficient at the surface, plays an important role in the temperature distribution. Also the temperature can be determined by knowing the laser power of the incident pulse and the pulse duration for the combinations listed in Table 2. By increasing the pulse duration, the temperature increasing monotonically. The grater anyone from the pulse duration time or the power of the incident laser pulse , or the two, the grater the thermal penetration depth. For small pulse duration the effect of the cooling factor is negligible

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